REVIEW OF UNDERGROUND CABLE IMPEDANCE AND ADMITTANCE FORMULAS

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ABSTRACT

Cable impedance and admittance formulas are essential to study steady-state and transient phenomena on a cable. Pollaczek derived the earth-return impedance of an underground cable in 1928. Wedepohl and Wilcox derived the cable internal impedance and admittance in 1973. Then, the formulation of the cable impedance and admittance matrices was generalized and implemented into well-known EMTP as a subroutine "Cable Constants" in 1976 by Ametani. Since then a number of transient simulations on cable systems have been carried out using the EMTP, and there are many papers discussing the above mentioned work and deriving new formulas, either accurate or approximate. This paper has reviewed and summarized the previous publications to give an idea of what are the cable impedance, admittance, and the EMTP simulations of the cable transients.

KEYWORDS

Cable; impedance; admittance; transient; EMTP.

I. INTRODUCTION

A number of underground cable transmission systems are under construction and/or are planned in many countries [1-3]. For the insulation design and coordination of an underground cable, it is essential to predict and investigate possible over-voltages. Cable impedance and admittance formulas are necessary to study transient and steady-state phenomena on the cable.

The impedance and admittance formulation of a cable is far more complicated than that of an overhead line, because even a single-phase cable consists of two conductors at least, i.e. a core conductor and a metallic sheath (shield) in the case of a single-core coaxial cable (SC cable) [4, 5]. Also a long high-voltage SC cable is quite often cross-bonded, similar to overhead line transposition. Furthermore, a so-called pipe-type cable (PT cable), such as a POF cable, is composed of three-phase cables installed within a conducting pipe. Then the PT cable becomes a seven-conductor system.

An impedance formula of a cylindrical conductor was derived by Schelkunoff in 1932 [6]. The impedance and admittance formulas of an SC cable were developed by Wedepohl and Wilcox [4]. The impedance and admittance formulas of a PT cable, where an inner conductor is eccentric to the pipe centre, were developed by Brown and Rocamora [7]. The earth-return impedance of an underground cable was derived by Pollaczek in 1926[8]. The formulas have been generalized and implemented into well-known EMTP (Electro-Magnetic Transients Program) as a subroutine "Cable Constants" in 1976 by Ametani in the Bonneville Power Administration, US Department of Energy [5, 9].

This paper summarizes and reviews the impedance and admittance formulation of three-phase SC and PT cables. Also, problems of the formulas and their applications are reviewed, and a recent trend of the cable impedance and admittance calculations is explained.

II. IMPEDANCE AND ADMITTANCE FORMULATION

A.Formulation of impedance and admittance

The Impedance and admittance of a cable system are defined in the two matrix equations [5].

$$d\,\mathbf{V}/dx = -\mathbf{Z}\cdot\mathbf{I} \qquad d\,\mathbf{I}/dx = -\mathbf{Y}\cdot\mathbf{V} \tag{1}$$

where $\mathbf{V}, \mathbf{I}$: voltage and current vectors at distance x,

$\mathbf{Z}, \mathbf{Y}$: square matrices of impedance and admittance.

In general, the impedance and admittance matrices of a cable can be expressed in the following forms [5].

$$\mathbf{Z} = \mathbf{Z}_i + \mathbf{Z}_p + \mathbf{Z}_c + \mathbf{Z}_0 \tag{2}$$

$$\mathbf{Y} = s\cdot\mathbf{P}^{-1},\ \mathbf{P} = \mathbf{P}_i + \mathbf{P}_p + \mathbf{P}_c + \mathbf{P}_0 \tag{3}$$

where $\mathbf{P}$ is a potential coefficient matrix and $s = j\omega$.

In the above equations, the matrices with subscripts "i" concern an SC cable and the matrices with subscript "p" and "c" are related to a pipe enclosure. The matrices with subscript "o" concern cable outer media, i.e. air space and earth. When a cable has no pipe enclosure, there exists no matrix with subscripts "p" and "c".

In the above formulation implemented in the EMTP, the following assumptions are made [5].

- The displacement currents and dielectric losses are negligible.
- Each conducting medium of a cable has constant permeability.
- The pipe thickness is greater than the penetration depth of the pipe wall for the PT cable case.

The details will be explained in the following sections.

B. Impedance matrix

B1. Internal impedance of a single-core coaxial cable (SC cable)

Assume that an SC cable consists of a core, sheath and armor as shown in Fig. 1(a). The impedance matrix is given in the following form.

$$\mathbf{Z_i} = \begin{bmatrix} \mathbf{Z}_{i1} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{Z}_{in} \end{bmatrix} \tag{4}$$

All the off-diagonal sub-matrices of $\mathbf{Z}_i$ are zero.

A diagonal sub-matrix $\mathbf{Z}_{ij}$ ($j = 1,\dots,n$ for an n-phase SC cable) expresses the self-impedance matrix of one phase SC cable, which is given by:

$$\mathbf{Z}_{ij}=\begin{bmatrix} Z_{ccj} & Z_{csj} & Z_{caj} \\ Z_{csj} & Z_{ssj} & Z_{saj} \\ Z_{caj} & Z_{saj} & Z_{aaj} \end{bmatrix} \quad (5)$$

Z_{ccj} = core self-impedance = $z_{cs}+z_{sa}+z_{a4}-2z_{2m}-2z_{3m}$ (6)

Z_{ssj} = sheath self-impedance = $z_{sa}+z_{a4}-2z_{3m}$ (7)

Z_{aaj} = armour self-impedance = z_{a4} (8)

Z_{csj} = core to sheath mutual impedance = $z_{sa}+z_{a4}-z_{2m}-2z_{3m}$ (9)

Z_{caj} = core to armour mutual impedance = $z_{a4}-z_{3m}$ (10)

Z_{saj} = sheath to armour mutual impedance = Z_{caj} (11)

$z_{cs}=z_{10}+z_{1in}+z_{2i},\ z_{sa}=z_{20}+z_{2in}+z_{3i},\ z_{a4}=z_{30}+z_{3in}$ (12)

where z_{k0}: outer surface impedance of conductor k

z_{km} : inner to outer surface impedance of conductor k

z_{ki} : inner surface impedance of conductor k

z_{kin} : outer insulator impedance of conductor k

and k=1 for core, k=2 for sheath, k=3 for armor.

When the SC cable consists of a core and sheath, the matrix of (5) is reduced to a 2X2 matrix.

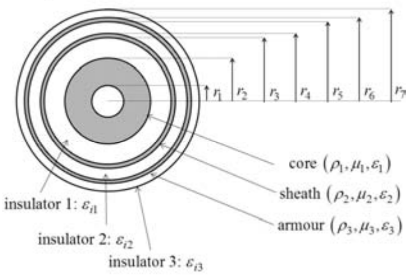
$$\mathbf{Z}_{ij}=\begin{bmatrix} Z_{ccj} & Z_{csj} \\ Z_{csj} & Z_{ssj} \end{bmatrix} \quad (13)$$

$$Z_{ccj}=z_{cs}+z_{s3}-2z_{2m},\ Z_{ssj}=z_{s3}$$
$$Z_{csj}=z_{s3}-z_{2m},\ z_{s3}=z_{20}+z_{23} \quad (14)$$

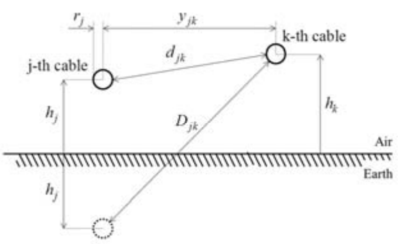
If an SC cable consists only of a core, the sub-matrix is reduced to one element, i.e.

$$\mathbf{Z}_{ij}=Z_{ccj}=z_{11}+z_{12} \quad (15)$$

The component impedances per unit length in the above equations are given in Appendix A.



(a) SC cable cross-section.



(b) System configuration.

Fig. 1 Single-core coaxial (SC) cable.

B2. Impedance matrices $\mathbf{Z_p}$ and $\mathbf{Z_c}$ of a pipe-type cable (PT cable)

The impedance matrix of a PT cable shown in Fig. 2, where an inner conductor is an SC cable, is given by:

(a) Pipe thickness assumed to be infinite

$$\mathbf{Z}=\mathbf{Z}_i+\mathbf{Z}_p \quad (16)$$

(b) Pipe thickness being finite

$$\mathbf{Z}=\mathbf{Z}_i+\mathbf{Z}_p+\mathbf{Z}_c+\mathbf{Z}_0 \quad (17)$$

$$\mathbf{Z}_p=\begin{bmatrix} \mathbf{Z}_p & \mathbf{0}_t \\ \mathbf{0} & 0 \end{bmatrix},\ \mathbf{Z}_c=\begin{bmatrix} \mathbf{Z}_c & \mathbf{Z}_{c2_t} \\ \mathbf{Z}_{c2} & Z_{c3} \end{bmatrix},\ \mathbf{Z}_i=\begin{bmatrix} \mathbf{Z}_i & \mathbf{0}_t \\ \mathbf{0} & 0 \end{bmatrix} \quad (18)$$

where $\mathbf{Z}_p$: pipe internal impedance, $\mathbf{z}_c$: inner to outer surface impedance, $\mathbf{z}_i$: SC cable impedance in a pipe.

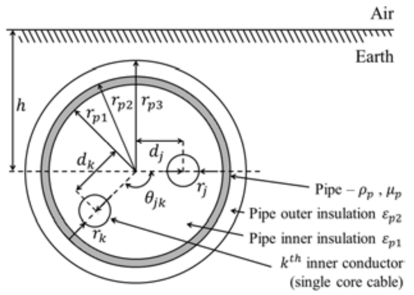


Fig. 2 A PT cable.

In (18), the last columns and rows correspond to the pipe conductor. Thus, these should be omitted when the pipe thickness is assumed as infinite. A diagonal sub-matrix of $\mathbf{Z}_i$, in (16) or (17), is given in (5). A sub-matrix of $\mathbf{Z}_p$, (18), is given in the following form.

$$\mathbf{Z}_{pjk}=Z_{pjk}\cdot\mathbf{U},\ \text{where } \mathbf{U}: 3\times 3 \text{ matrix of 1's} \quad (19)$$

When an inner conductor consists of a core and a sheath, (19) is reduced to a 2X2 matrix. When the inner conductor consists only of a core, (19) is further reduced to a column matrix in the same manner as explained in the case of $\mathbf{Z}_i$ in Section B1. This is the same for all the other impedance and admittance matrices explained in this section. The formula of Z_{pjk} is given in Appendix B1. A sub-matrix and the last row and column elements of $\mathbf{Z_c}$ in (1) are given in the following form:

$$\mathbf{Z}_{c1}=\begin{bmatrix} Z_{c1} & Z_{c1} & Z_{c1} \\ Z_{c1} & Z_{c1} & Z_{c1} \\ Z_{c1} & Z_{c1} & Z_{c1} \end{bmatrix},\quad \begin{matrix} Z_{c1}=Z_{c3}-2z_{pm} \\ Z_{c2}=Z_{c3}-z_{pm} \\ Z_{c3}=z_{p0}+z_{p3} \end{matrix} \quad (20)$$

The formulas of z_{p0}, z_{pm}, z_{p3} are given in Appendix B2.

B3. Impedance matrix $\mathbf{Z_0}$ of cable outer medium: earth-return impedance

The cable outer medium impedance matrix $\mathbf{Z}_0$ is given in the following form in general.

$$\mathbf{Z}_0=\begin{bmatrix} \mathbf{Z}_0 & \mathbf{Z}_{0t} \\ \mathbf{Z}_0 & Z_0 \end{bmatrix} \quad (21)$$

where sub-matrix $\mathbf{z}_o$ is given by

$$\mathbf{Z}_o = \begin{bmatrix} \mathbf{Z}_{011} & \mathbf{Z}_{012} & ... & \mathbf{Z}_{01n} \\ \mathbf{Z}_{012} & \mathbf{Z}_{022} & ... & \mathbf{Z}_{02n} \\ ... & ... & ... & ... \\ \mathbf{Z}_{01n} & \mathbf{Z}_{02n} & ... & \mathbf{Z}_{0nn} \end{bmatrix} \quad (22)$$

A sub-matrix of the earth return impedance $\mathbf{Z}_0$ in (22) is given in the following form.

$$\mathbf{Z}_{0jk} = Z_{0jk} \cdot \mathbf{U} \quad (23)$$

$\mathbf{Z}_{0jk}$ in (23) is the earth-return impedance between the j-th and the k-th cables. When a cable system is overhead, the impedance is given by Carson [10]. When a system is underground, the impedance is given by Pollaczek[8]. If an earth is stratified, the earth-return impedance derived by Nakagawa et al. can be used [11].

A diagonal sub-matrix of $\mathbf{Z}_{0jk}$ in (23) for a PT cable is :

$$\mathbf{Z}_{0jk} = Z_0 \cdot \mathbf{U} \quad (24)$$

Z_0 in the above matrix is the self-earth-return impedance of the pipe given in Appendix B3.

C. Potential coefficient matrix

The admittance matrix of a cable system is evaluated from the potential coefficient matrix as given in (3).

C1. SC cable internal potential coefficient $\mathbf{P}_i$

In the SC cable case, $\mathbf{P}_p$ and $\mathbf{P}_c$ are zero, and when the cable system is underground, $\mathbf{P}_0$ is also zero. The internal potential coefficient matrix $\mathbf{P}_i$ is given by:

$$\mathbf{P}_i = \begin{bmatrix} \mathbf{P}_{i1} & 0 & ... & 0 \\ 0 & \mathbf{P}_{i2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \mathbf{P}_{in} \end{bmatrix} \quad (25)$$

All the off-diagonal sub-matrices of $\mathbf{P}_i$ are zero. A diagonal sub-matrix expresses the potential coefficient matrix of an SC cable. When the SC cable consists of a core, sheath and armor as shown in Fig. 1(a), the diagonal sub-matrix is given in the following form.

$$\mathbf{P}_{ij} = \begin{bmatrix} P_{cj}+P_{sj}+P_{aj} & P_{sj}+P_{aj} & P_{aj} \\ P_{sj}+P_{aj} & P_{sj}+P_{aj} & P_{aj} \\ P_{aj} & P_{aj} & P_{aj} \end{bmatrix} \quad (26)$$

$$P_{cj} = \frac{1}{2\pi\varepsilon_0\varepsilon_{i1}} \ln\frac{r_3}{r_2},\ P_{sj} = \frac{1}{2\pi\varepsilon_0\varepsilon_{i2}} \ln\frac{r_5}{r_4},\ P_{aj} = \frac{1}{2\pi\varepsilon_0\varepsilon_{i3}} \ln\frac{r_7}{r_6} \quad (27)$$

C2. Potential coefficients of a PT cable

The potential coefficient(P-C) matrix of a PT cable shown in Fig. 2 is given in the following form.

(1) Pipe thickness assumed to be infinite

$$\mathbf{P} = \mathbf{P}_i + \mathbf{P}_p \quad (28)$$

(2) Pipe thickness being finite

(a) Underground cable: $\mathbf{P} = \mathbf{P}_i + \mathbf{P}_p + \mathbf{P}_c$ (29)

(b) Overhead cable: $\mathbf{P} = \mathbf{P}_i + \mathbf{P}_p + \mathbf{P}_c + \mathbf{P}_0$ (30)

$$\mathbf{P}_i = \begin{bmatrix} \mathbf{P}_i & \mathbf{0}_t \\ \mathbf{0} & 0 \end{bmatrix},\ \mathbf{P}_p = \begin{bmatrix} \mathbf{P}_p & \mathbf{0}_t \\ \mathbf{0} & 0 \end{bmatrix},\ \mathbf{P}_c = \begin{bmatrix} \mathbf{P}_c & \mathbf{P}_{c_t} \\ \mathbf{P}_c & P_c \end{bmatrix} \quad (31)$$

where $\mathbf{P}_i$: SC cable internal P-C, $\mathbf{P}_p$: pipe internal P-C, $\mathbf{P}_c$: pipe inner to outer surfaces P-C.

In (33), the last columns and rows correspond to the pipe conductor. These should be omitted when the pipe thickness being assumed infinite. In (33), a diagonal sub-matrix of $\mathbf{P}_i$ is given in (27) and a sub-matrix $\mathbf{P}_{pjk}$ of $\mathbf{P}_p$ is given in the following form.

$$\mathbf{P}_{pjk} = P_{pjk} \cdot \mathbf{U} \quad (32)$$

$\mathbf{P}_{pjk}$ in the above equation is the potential coefficient between the j-th and the k-th inner conductors with respect to the pipe inner surface, and is given in the following form:

$$P_{p_{jj}} = Q_{jj} \big/ 2\pi\varepsilon_0\varepsilon_{p1},\ P_{p_{jk}} = Q_{jk} \big/ 2\pi\varepsilon_0\varepsilon_{p1} \quad (33)$$

Q_{jj} and Q_{jk} are given in Appendix B1.

A sub-matrix and the last column and row elements of $\mathbf{P_c}$ in (33) are given by:

$$\mathbf{P}_c = P_c \cdot \mathbf{U} \quad (34)$$

C3. Potential coefficient of a cable in air $\mathbf{P}_0$

The potential coefficient matrix $\mathbf{P}_0$ is given, in general in the following form:

$$\mathbf{P}_0 = \begin{bmatrix} \mathbf{P}_0 & \mathbf{P}_{0t} \\ \mathbf{P}_0 & 0 \end{bmatrix} \quad (35)$$

In the case of an SC cable, there is no last column and row, i.e.

$$\mathbf{P}_0 = [\mathbf{P}_0] \quad (36)$$

The sub-matrices of $\mathbf{P}_0$ are given in the following form.

$$\mathbf{P}_{0jk} = P_{0jk} \cdot \mathbf{U} \quad (37)$$

where $\mathbf{P}_{0jk}$ is the space potential coefficient and is given for the case of Fig. 1(b) by:

$$P_{0jj} = \frac{1}{2\pi\varepsilon_0} \cdot \ln\left(\frac{2h_j}{r_{7j}}\right),\ P_{0jk} = \frac{1}{2\pi\varepsilon_0} \cdot \ln\left(\frac{D_{jk}}{d_{jk}}\right) \quad (38)$$

III. PROBLEMS AND RECENT TREND OF IMPEDANCE AND ADMITTANCE CALCULATIONS

A. Problems of existing formulas and EMTP Cable Constants

A1. Earth-return impedance of underground cables

It is well-known that Pollaczek's earth-return impedance formula of underground cables [8] is numerically very unstable. Although Wilcox derived a series expansion form of Pollaczek's formula [4], similar to Carson's one for an overhead line [10], there are many mistypes in the paper. Because of the facts, Ametani made the following approximation in the exponential term of Pollaczek's earth-return impedance for underground cables.

$$\exp\left\{-\left|h_i+h_j\right|\sqrt{s^2+m_1^2}\right\}\rightarrow\exp\left\{-\left|h_i+h_j\right|s\right\} \quad (39)$$

Then the integral part becomes the same as the earth-return impedance of an overhead line derived by Pollaczek and Carson [8, 10]. By adopting the infinite series expansion derived by Carson, the earth-return impedance of the underground cable is easily calculated, and no numerical instability arises. Thus, this has been implemented into the EMTP Cable Constants since 1976. However, the accuracy of the approximation decreases in a high frequency region.

The numerical instability observed in 1970s to 1990s can be overcome by the advanced numerical calculation technology, and thus it is expected to remove the above approximation in the present Cable Constants.

A2. The number of PT cables

Occasionally there was a request to increase the number of PT cables to deal with multi-circuit gas-insulated buses and/or multi-circuit POF cables, for which each bus and/or cable is a PT cable with three-phase inner conductors. Theoretically, it is straightforward to re-write Z_p and Z_c in (16) and (17) for a multi-circuit PT system, i.e.

$$\mathbf{Z}=\begin{bmatrix}\mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots\\ \mathbf{Z}_{12} & \mathbf{Z}_{22} & \cdots\\ \cdots & \cdots & \cdots\end{bmatrix}$$

The terms $\mathbf{Z}_{ii}$ are given as $\mathbf{Z}$ in (18), and the mutual impedance between PT cables i and j is:

$$\mathbf{Z}_{ij}: \text{n×n matrix}$$

However, this necessitates complete rewriting of the Cable Constants which might require an enormous man-power.In reality, a multi-circuit PT cable system composed of n PT cables can be dealt with as n independent PT cables because the pipe wall is, in general, thick enough and thus the mutual coupling between the i-th and the j-th PT cable does not influence the pipe inner conductors.

A3. Pipe outer surface impedance

In the PT cable formulation in EMTP Cable Constants, the outer surface impedance of a coaxial cable is adopted as the pipe outer surface impedance, because no impedance formula for the pipe outer surface was developed in 1970s. Occasionally, it was criticized that the pipe outer surface impedance was not accurate. To respond the criticism, a complete solution of the PT cable has been derived by Amekawa et al. [12]. Thus, the impedance should be replaced by the complete formula.

A4. Semiconducting layer

There were many mails asking how to deal with the semiconducting layer of a cable and requesting to make the Cable Constants possible to deal with the semiconducting layers in the late 1990s from European EMTP users. To respond the request, the impedance formula of the semiconducting layer has been developed [13]. The formula is given as a combination of existing formulas in the Cable Constants. Therefore, it is not necessary to implement the formula into Cable Constants. A user can use the Cable Constants to calculate the component impedances z_i, z_m and z_0. Then, by Excel, Matlab, etc., the semiconducting layer impedance is easily obtained in the following equation [13].

$$Z_{semic}=z_{20}-z_{2m}^2/\left(z_{10}-z_{2i}\right) \quad (40)$$

A5. Frequency-dependent permittivity

There were too many requests to implement the frequency-dependent permittivity into the Cable Constants. This is theoretically very easy. That is, modify the code of the admittance calculation as follows:

$$\varepsilon(\omega)=\varepsilon_r(\omega)+j\varepsilon_i(\omega),\ Y(\omega)=G(\omega)+j\omega C(\omega) \quad (41)$$

However, Ametani refused the requests. The reason is simple. A user can hardly obtain the frequency-dependent permittivity which is complex as in the above equation. He has collaborated with many cable manufacturers, but it has been impossible to obtain a complete frequency-dependent $\varepsilon_r(\omega)$ and $\varepsilon_i(\omega)$ or $\tan\delta(\omega)$ which cover the frequencies required for a cable transient simulation. Further, it should be noted that the temperature dependence of the permittivity is very heavy and can be far more significant than the frequency dependence. Table 1 shows some examples of the frequency/temperature dependent permittivity [14].

Table 1. Temperature and frequency dependent permittivity and loss factor

Material	T (ºC)	f (Hz)	ε_i	$\tan\delta$
Water	1.5	10^5	87	1900
		10^8	87	70
	25	10^5	78.2	4000
		10^8	78	50
	85	10^5	58	12400
		10^8	58	30
Paper	25	10^2	3.3	58
		10^5	3.1	200
		10^8	2.77	660
	82	10^2	3.57	170
		10^5	3.4	85
		10^8	3.08	680

Ametani has never been supplied such data from EMTP users who have requested the frequency-dependent permittivity. As one of the original developers of the EMTP, he has experienced too many requests, comments and criticisms about the EMTP and the Cable Constants.

It should be reminded that an underground cable and an overhead line are isolated from a conducting medium, while a grounding electrode is a bare conductor so that a fault or lightning current is flowing into the ground. The Line Constants and the Cable Constants of the EMTP are for the isolated/insulated conductor, but not for the bare conductor connected to or touching a conducting medium. However, it is easy to implement the bare conductor option to the Cable Constants by adopting well-known formulas of the grounding electrodes by Sunde [15].

A6. Proximity effect

There are a number of publications discussing the proximity effect in a cable, especially in an SC cable of which the core conductor is composed of a number of stranded wires, sectioned as three or four fan-shaped blocks [1]. Stranded conductors are also used quite often in high-voltage overhead lines. Further, each stranded conductor may be enamel coated, i.e. isolated from the

other strands. Also, in a PT cable, the inner phase conductors (SC cables) are installed at the bottom of the pipe, and thus phase cables touch each other.
Under the above situation, the proximity effect might cause a significant effect on the attenuation, the propagation velocity and a resultant transient voltage [16]. The attenuation increases and the voltage decreases as the eccentricity (or the proximity between cable phase and pipe) becomes large. As a result, a transient voltage waveform significantly differs from that with no eccentricity.
If there is an accurate formula for internal impedance of a conductor considering the proximity effect, then the formula can be implemented by replacing the present impedance formulas.

A7. Spiral wire

It is assumed in the Cable Constants that all the cable conductors, such as a metallic sheath and an armour conductor, are of cylindrical circular shape. However, in a real cable, the metallic sheath is often a wire but not a cylinder. Also, a steel wire is often used as the armour. Because there is no accurate impedance formula for the self and mutual impedance of such a spiral wire, the wire has been represented by a circular cylinder of which the volume of the wire is the same as that of the cylinder [17],

$$S_w \cdot l_w = \pi\left(r_o^2 - r_i^2\right) \cdot l \qquad (42)$$

where S_w and l_w are the cross-section and length of the wire and r_o, r_i and l are the outer and inner radius, and the length of the cylinder.
If there are accurate formulas of the wire self-impedance considering the spiral, and the mutual impedance between the cylindrical core and the spiral wire, the present equivalent impedance assuming a cylindrical sheath should be replaced.

B. Numerical Electromagnetic Analysis

A numerical electromagnetic analysis (NEA) is becoming a powerful tool to analyze an electromagnetic phenomena both in a steady-state and in a transient [18]. The NEA is a numerical approach to solve Maxwell's equations directly based on the physical and geometrical parameters of given media together with given boundary conditions.
NEC (Numerical Electromagnetic Code) is a typical example of software based on numerical electromagnetic analysis used for studying electromagnetic phenomena in frequency domain. Those software are based on the Finite Elements Method (FEM) and on Method of Moments (MoM). Another powerful and widely used approach is Finite Difference Time Domain (FDTD) method which solves Maxwell's equation in time domain, and thus the FDTD method is quite popular for transient studies.
In this section, the NEC and the FDTD method are explained briefly as a tool to calculate wave propagation related parameters and transient voltage/current waveforms on a distributed-parameter circuit such as a cable.

B1. NEC

NEC has been developed in the Lawrence Livermore National Laboratory. Because a conductor is represented by a combination of cylindrical segments, the NEC can easily deal with a conductor with an arbitrary cross-section and also with a thin wire which causes a difficulty in an FDTD method [18]. However, the number of segments to represent a conductor with a given length becomes very large when the frequency of an applied current or voltage source is high. The segment length Δx should be much smaller than the wave length λ. For example, $f = 50\,\mathrm{MHz}$,

$$\lambda = c_0 / f = 3\times10^8 / 50\times10^6 = 6\ \mathrm{m}$$

If the conductor length is $x = 3000\ \mathrm{m}$, the number of the segments is taken as 500X20 with $\Delta x = \lambda/20$ to satisfy the condition of Δx being much smaller than λ to ensure the accuracy of a simulation using NEC. Therefore, the required computer memory and computation time becomes very large.

B2. FDTD

In an FDTD method, a three-dimensional (x, y, z) space and the time are discretized to solve Maxwell's equation under given boundary conditions. The discretized elements are called "cells". The dimensions of the analytical space including a conductor, air and earth must be large enough and the cell size small enough for an accurate simulation, similarly to the segment length in NEC. Assume the following case: an overhead conductor of length $x = 1\ \mathrm{m}$, height $h = 6\ \mathrm{cm}$ and radius $r = 1\ \mathrm{mm}$ and an observation time $T = 50\ \mathrm{ns}$. Then to ensure the FDTD simulation accuracy, the following conditions are necessary:

$$X = 1.2\,\mathrm{m}, Y = Z = 18\,\mathrm{cm},$$
$$\Delta s = \Delta x = \Delta y = \Delta z = 1\,\mathrm{mm}, \Delta t = 1.925\,\mathrm{ps}$$

It is clear that the memory required for the analytical space becomes $1.2\times10^3\times180\times180 = 39\times10^6$ and the number of calculations is $T/\Delta t = 26\times10^3$. This simulation takes altogether about 5 hours using a powerful PC (Epson Intel Core i7 Extreme).

B3. Problems of numerical electromagnetic analysis

As it is clear from the principle of the NEA explained above, NEA methods require a large amount of computer memory and CPU time. In the NEC, the total number of memories is dependent on the number of segments, which is proportional to the segment length and the conductor length. The higher the frequency, the smaller the segment length. To ensure the numerical accuracy of the NEC, a small segment length is essential.
In an FDTD method, required computer memory is proportional to the size of the analytical space, and inversely proportional to the cell size which is, in theory, to be smaller or equal to the conductor radius. Its numerical accuracy is significantly affected by the size of the analytical space, the cell size and the computation time step.

IV. CONCLUSIONS

The impedance and admittance formulation of a single-core coaxial (SC) and pipe-type (PT) cables implemented in the EMTP Cable Constants are summarized together with the formulas which are far more complicated than those of overhead lines.

Problems related to the formulation and the impedance formulas have been reviewed, and possible countermeasures are described. Some of the problems come from approximations adopted in the formulation and assumptions made to derive the impedance formulas.

To overcome these problems, a numerical electromagnetic analysis such as NEC and an FDTD method are explained. By adopting the numerical electromagnetic analysis, many of the problems reviewed in the paper can be solved. However, this requires a large amount of computer memory and computation time. The accuracy is heavily dependent on the memory size, the segment length in the NEC, the cell size and the time step in the FDTD.

APPENDIX: IMPEDANCE FORMULAS

A. SC Cable Internal Impedance

z_o=conductor outer surface impedance

$$=(s\mu_0\mu_1/2\pi)\cdot(1/x_2D_1)\cdot\{I_0(x_2)\cdot K_1(x_1)+K_0(x_2)\cdot I_1(x_1)\} \quad \text{(A.1)}$$

z_{in}=conductor outer insulator impedance

$$=(s\mu_0\mu_{i1}/2\pi)\cdot\ln(r_3/r_2) \quad \text{(A.2)}$$

z_i=conductor inner surface impedance

$$=(s\mu_0\mu_1/2\pi)\cdot(1/x_1D)\cdot\{I_0(x_1)\cdot K_1(x_2)+K_0(x_1)\cdot I_1(x_2)\} \quad \text{(A.3)}$$

$$z_m=\frac{\rho_2}{2\pi r_2 r_3 D}=\text{inner to outer surface impedance} \quad \text{(A.4)}$$

$$\begin{aligned} &D=I_1(x_2)\cdot K_1(x_1)-I_1(x_1)\cdot K_1(x_2),\\ &x_k=\beta_k\sqrt{s},\ \beta_2=r_2\sqrt{\mu_0\mu_1/\rho_1},s=j\omega \end{aligned} \quad \text{(A.5)}$$

B. PT cable

B1. Z_{pjk}

$$\begin{aligned} Z_{pjk}&=(s\mu_0/2\pi)\cdot\Big[\mu_pK_0(x_1)/\mu_1K_1(x_1)+Q_{jk}+\\ &+2\mu_p\sum_{n=1}^{\infty}C_n\Big/\big\{n(1+\mu_p)+x_1K_{n-1}(x_1)/K_n(x_1)\big\}\Big] \end{aligned} \quad \text{(A.6)}$$

$$\left.\begin{aligned} &Q_{jj}=\ln\left[(r_{p1}/r_j)\cdot\left\{1-(d_j/r_{p1})^2\right\}\right]\\ &Q_{jk}=\ln\left[r_{p1}\Big/\sqrt{d_j^2+d_k^2-2d_jd_k\cos\theta_{jk}}\right]-\sum_{n=1}^{\infty}\frac{C_n}{n}\\ &C_n\left(d_jd_k/r_{p1}^2\right)^n\cdot\cos(n\theta_{jk}) \end{aligned}\right\} \quad \text{(A.7)}$$

B2. Z_c

$$\left.\begin{aligned} &z_{pm}=z_m \text{ in (A.4)}, z_{p0} \text{ in (A.1)}\\ &z_{p3}=z_{in} \text{ in (A.2)} \end{aligned}\right\} \quad \text{(A.8)}$$

B3. Z_e

$$\begin{aligned} &Z_{e_{ij}}=j\omega\frac{\mu_0}{2\pi}\left[K_0(md_{ij})-K_0(mD_{ij})+2\int_0^{\infty}\frac{F(s)}{|s|+\sqrt{s^2+m^2}}ds\right]\\ &F(s)=\exp\left\{-|h_i+h_j|\sqrt{s^2+m^2}\right\}\cos(y_{ij}\cdot s) \end{aligned} \quad \text{(A.9)}$$

Overhead Cable

$$\left.\begin{aligned} &Z_{e_{ij}}=j\omega\frac{\mu_0}{2\pi}[P_0+Q-jR],\ P_0=\ln(D_{ij}/d_{ij})\\ &Q-jR=2\int_0^{\infty}\frac{F(s)}{1+s}ds,\ F(s)=\exp\{-(h_i+h_j)s\}\cos(y_{ij}\cdot s)\\ &D_{ij}^2=(h_i+h_j)^2+y_{ij}^2,\ d_{ij}^2=(h_i-h_j)^2+y_{ij}^2 \end{aligned}\right\} \quad \text{(A.10)}$$

REFERENCES

[1] CIGRE WG B1.30, “Cable Systems Electrical Characteristics, CIGRE TB-531, Apr. 2013.

[2] CIGRE WG C4.502, "Power System Technical Performance Issues Related to the Application of Long HVAC Cables," CIGRE TB-556, Oct. 2013.

[3] www.energinet.dk,"Cable Action Plan", March 2009.

[4] L. M. Wedepohl and D. J. Wilcox, *"Transient analysis of underground power transmission systems : system model and wave propagation characteristics," Proc. IEE,* vol. 120, no.2, pp.253-260, 1973.

[5] A. Ametani, “General formulation of impedance and admittance formulas”, IEEE Trans. PAS, vol. PAS-99, no. 3, pp.902-910, May/June 1980.

[6] S. A. Schelkunoff, "The electromagnetic theory of coaxial transmission line and cylindrical shields", Bell Syst. Tech. J., vol.13, pp.532-579, 1934.

[7] G. W. Brown and R. G. Rocamora, "Surge propagation in three-phase pipe-type cables, Part I-Unsaturated pipe", IEEE Trans. PAS., vol.PAS-95, pp.89-95, 1976.

[8] F. Pollaczek, "Uber das Feld einer unendlich langen wechsel stromdurchflossenen Einfachleitung",E.N.T., Band 3 (Heft 9), pp.339-360, 1926.

[9] W. Scott-Meyer, “EMTP Rule Book”, Bonneville Power Administration, Portland, Ore., 1978:

[10] J. R. Carson, "Wave propagation in overhead wires with ground return", Bell Syst. Tech. Journal., vol.5, pp.539-554, 1926.

[11] N. Nakagawa, A. Ametani and K. Iwamoto,.”Further studies on wave propagation in overhead lines with earth-return : impedance of stratified earth”, Proc.IEE, vol.120, no.12, pp.1521-1528, 1973

[12] N. Amekawa, N. Nagaoka and A. Ametani, “Impedance derivation and wave propagation characteristics of pipe-enclosed and tunnel-installed cables”,IEEE Trans. PWRD, vol. 19, no. 1, pp/ 380-386, Jan. 2004.

[13] A. Ametani, Y. Miyamoto and N. Nagaoka, “Semi-conducting layer impedance and its effect on cable wave propagation and transient characteristics”, IEEE Trans. PWRD, vol. 19, pp. 1523-1531, 2004.

[14] A. Ametani, “Further improvements of cable constants and an investigation of cable problems, EMTP News, vol. 1, no. 4, pp. 4-13, Dec. 1988.

[15] E. D. Sunde, “Earth Conduction Effects in Transmission Systems”, Dover, New York, 1968.

[16] A. Ametani, et al, “Wave propagation characteristics on a pipe-type cable in particular reference to the proximity effect”, IEEJ Trans. PE, vol. 133, no. 2, pp. 954-960, 2013.

[17] A. Ametani, and I. Fuse, “Approximate method for calculating the impedance of multi-conductor with arbitrary cross-section“, Electrical Eng. Japan, vol. 112, no. 2, pp. 117-123, 1992.

[18] CIGRE WG C4.501, "Guideline for Numerical Electro-magnetic Analysis Method and its Application to Surge Phenomena", CIGRE TB-543, June 2013.