



## D.2.2. Répartition du champ électrique dans un câble coaxial à courant continu

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### Sommaire :

La température d'un câble à courant continu croît depuis la gaine extérieure jusqu'au conducteur chauffé par effet Joule.

La prédiction de la distribution radiale du champ électrique est nécessaire pour un dimensionnement fiable du câble.

La conductivité  $\sigma$  de l'isolation croît à la fois avec la température et le champ électrique local. La détermination exacte de la distribution de champ résulte de calculs auto cohérents qui ne sont pas évidents, même si la variation de  $\sigma$  avec le champ électrique est approximée par des fonctions simples.

Dans ce travail,  $\sigma$  est supposé suffisamment petit en tout point de l'isolant pour que l'échauffement de celui-ci par effet Joule n'affecte pas le profil de température. Cette hypothèse a été discutée par Eoll.

A partir des équations décrivant la conservation radiale du courant, le champ électrique est d'abord calculé dans l'hypothèse où  $\sigma$  dépend seulement de la température, puis dans le cas où  $\sigma$  dépend également du champ électrique selon une loi exponentielle, ou selon le modèle de POOLE-FRENKEL.

Les résultats relatifs aux deux modèles de variation de  $\sigma$  en fonction du champ électrique sont discutés.

### 1) THE RADIAL TEMPERATURE PROFILE :

We assume that the conductivity of the insulation is so small everywhere in the insulation, even near the heated core, that JOULE heating is much too small to affect the temperature profile appreciably. We assume also that the variation of the material thermal conductivity and of the cable dimension due to the temperature gradient can be neglected.

The radial heat flux does not depend on the radius  $r$  and the above assumptions, combined with the uniform heat conduction hypothesis, yields the differential equation for the steady-state temperature profile :

$$dT = C \frac{dr}{r} \quad (1)$$

where  $C$  is a constant with the dimension of a temperature.

Integrating this between the inner radius  $r_i$ , where the temperature is  $T_i$  and the external radius  $r_e$  where the temperature  $T_e$ , one readily obtains the temperature profile :

$$T(r) = T_e + (T_i - T_e) \frac{\ln(r_e / r)}{\ln(r_e / r_i)} \quad (2)$$

which verifies the conditions  $T(r_i) = T_i$  and  $T(r_e) = T_e$ .

This result is well known but it is worth adding that the temperature at the mean radius  $\bar{r} = \sqrt{r_e \cdot r_i}$  is  $(T_i + T_e)/2$ .

## D.2.2. On the field distribution in a coaxial DC power cable

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### Summary :

In a power D.C. cable under operation the temperature increases from the outer sheath at ambient temperature to the inner conductor which is heated by conduction from the warm current carrying conductor.

An estimation of the radial field distribution is required to predict the maximum applicable voltage to the cable so that the local field remains everywhere less than a safe fraction of the dielectric strength.

The conductivity  $\sigma$  of the insulation increasing with both the temperature and the local field, the actual field distribution results from a self-consistent relation which is not trivial, even if simple approximations are used for the temperature and field dependances of  $\sigma$ .

In this work,  $\sigma$  is assumed to remain so small everywhere in the insulation that Joule losses do not affect the temperature profile, which is calculated with a uniform heat conductivity. The validity of this assumption was discussed by Eoll.

Using the radial current conservation relation, the field is first calculated assuming that  $\sigma$  depends only on temperature, and the results confirms earlier findings. Then, the field dependence of  $\sigma$  is accounted for by a limited expansion of both a standard exponential, and the POOLE-FRENKEL formula. Finally, the various distributions are compared.

### 2) THE RADIAL FIELD DISTRIBUTION WITH $\sigma = \sigma(T), \forall r$ :

The most usual - and currently used - variation of  $\sigma$  with temperature is an Arrhenius type law of the form :

$$\sigma(T) = \sigma_\infty \exp\left(-\frac{U}{kT}\right) \quad (3)$$

where  $T$  is the absolute temperature and  $U$  an activation energy.

However, this equation gives rise to mathematical complications which can be avoided by using Wagner's approximation.

$$\sigma(T) = \sigma(T_0) \exp\left(\frac{T - T_0}{\theta}\right) \quad (4)$$

where  $\theta$  is of the order of  $8^\circ\text{C}$  for  $U=1\text{eV}$ . This approximation has been shown to be an adequate substitute to the Arrhenius law [1] so long as  $T$  remains "relatively" close to  $T_0$ . For a power cable in operation,  $T_e - T_i \leq 50^\circ\text{C}$ , so that the relative temperature difference  $\Delta T / T_0 \leq 0,15$  justifies the use of Wagner's approximation.

Introducing the temperature profile calculated above into Wagner's approximation (4) gives for  $\sigma(r)$  :

$$\sigma(r) = \sigma_e \exp\left[\frac{T_i - T_e}{\theta \cdot \ln(r_e / r_i)} \ln\left(\frac{r_e}{r}\right)\right] \quad (5)$$