# Global basis functions method for the efficient computation of power losses in submarine three-core cable armor

Luca **GIUSSANI**, Luca **DI RIENZO;** Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Milan, Italy, luca.giussani@polimi.it, luca.dirienzo@polimi.it

Massimo **BECHIS**, Prysmian S.p.A., Milan, Italy, massimo.bechis@prysmiangroup.com

Carlo **DE FALCO**, MOX—Modeling and Scientific Computing, Dipartimento di Matematica, Politecnico di Milano, Milan, Italy, carlo.defalco@polimi.it

## *ABSTRACT*

*An efficient method is proposed for the computation of electromagnetic losses in an array of parallel magnetic and conductive round wires placed in an arbitrary time harmonic transverse magnetic field. The method uses global basis functions derived from a series expansion of the exact solution for the case of a single wire. The method is well-suited to the computation of armor losses in 2D models of high-voltage submarine cables. The accuracy is verified by comparing the results with a finite element solution.*

## *KEYWORDS*

HV submarine cables, armor losses, spectral methods

## INTRODUCTION

The recent development of offshore wind farms has led to a strong interest in modeling the electromagnetic behavior of three-core submarine cables, and, in particular, in the evaluation of the losses induced in their steel wire armor. It is widely acknowledged that the commonly adopted IEC 60287 standard substantially overestimates the armor losses. More accurate estimates would be very helpful during the design phase. However the simulation of submarine three-core cables by means of traditional computational methods, such as the Finite Element Method (FEM), requires large computational resources, due to the three-dimensional character of the problem, the multi-scale geometry of the armor wires, and the skin effect. There is therefore a strong interest in developing alternative computational methods which could lower the computational burden of the simulations.

The method we propose can be classified as a spectral method. The main idea is that the approximate solution of the problem is represented as a linear combination of a certain number of smooth global basis functions which are non-zero over the whole space. If these basis functions are chosen in an appropriate way, then only a very small number of them is required to represent the solution with sufficient accuracy. This, in turn, implies that the number of unknowns of the problem (one complex coefficient for each basis function) is very small, thus reducing the computational complexity of the solution.

The paper is organized as follows. First, following the development of [1], the exact solution is presented for the case of a single, straight and infinitely long magnetic and conductive wire, placed in an arbitrary tangential magnetic vector field. The solution is given in terms of the coefficients of the expansion of the magnetic vector potential around the center of the wire.

The next section shows how the aforementioned coefficients can be numerically computed by sampling the vector potential along the boundary of the wire and performing a discrete Fourier transform, which can be implemented in an efficient way using the fast Fourier transform algorithm.

These developments lead in a straightforward way to the numerical solution of the problem in the case of a single isolated wire. The solution in the case of multiple wires, typical of cable armors, is slightly more involved and requires the solution of a linear system of equation, whose structure is presented in the text.

Finally the method is tested on a geometry which is representative of HV three-core submarine cables. The losses and the induced current density are evaluated in the armor and the results are compared with a finite element simulation, showing good agreement.

## EXACT SOLUTION FOR A SINGLE WIRE

We consider a single infinitely long cylindrical wire of radius $R$, relative magnetic permeability $\mu_r$ and conductivity $\sigma$, whose axis coincides with the $z$-axis of a cylindrical coordinate system $(r, \phi, z)$. The cylinder is exposed to an external time-harmonic transverse magnetic field $\mathbf{B}_0 = \nabla \times \mathbf{A}_0$ of angular frequency $\omega$. We suppose that $\mathbf{B}_0$ does not vary along the $z$-direction. Under these assumptions the problem is two-dimensional and the magnetic potential has a $z$-component only

$$\mathbf{A} = A_z(r, \phi)\hat{\mathbf{z}} \tag{1}$$

Inside the cylinder ($r \le R$), $A_z$ satisfies Helmoltz's equation

$$\nabla^2 A_{\text{int}} - m^2 A_{\text{int}} = 0 \tag{2}$$

where

$$m = \frac{1}{\delta} + j\frac{1}{\delta}, \quad \delta = \sqrt{\frac{2}{\omega\mu_r\mu_0\sigma}} \tag{3}$$

Outside the cylinder ($r \ge R$), $A_z$ satisfies Laplace equation

$$\nabla^2 A_{\text{ext}} = 0 \tag{4}$$

We can decompose the external vector field as

$$A_{\text{ext}} = A_0 + A' \tag{5}$$

where $A_0$ is the source potential, and $A'$ is the reaction field. Far away from the cylinder the reaction field should vanish

$$\lim_{r\to+\infty} A'(r, \phi) = 0, \quad \forall\phi \in [0, 2\pi] \tag{6}$$

On the surface of the cylinder ($r = R$) the following continuity conditions should hold

$$A_{\text{int}} = A_{\text{ext}} \tag{7}$$